Summit Public Schools
Summit, New Jersey

Grade Level / Content Area: 9-11 / Mathematics
Length of Course: 1 Year

Algebra 2/Trigonometry Honors

Course Description:

The algebraic properties of the real number system, equations and inequalities of the first and second degree, and elementary functions are reviewed. The real number system is extended to the complex number system. Conic sections are given geometric application. Coordinate geometry; systems of equations; and exponential, logarithmic, polynomial, trigonometric, and circular functions are studied in detail. Matrices and probability are covered as time permits. Students will receive a graphing calculator for their use throughout the course. They are responsible for the calculator just as for their text.

Course Pacing:

1. Preliminary Topics 8 days
2. Functions and Relations 8 days
3. Linear Functions, Equations, Inequalities, and Systems 22 days
4. Quadratic Functions and Complex Numbers; Quadratic Relations and Systems 30 days
5. Exponential and Logarithmic Functions 16 days
6. Rational and Irrational Algebraic Functions 24 days
7. Higher-Degree Functions 14 days
8. Trigonometric Ratios and Circular Functions 36 days
9. Probability and Data Analysis 10 days
10. Midterm Review and Exam 6 days
11. Final Review and Exam 6 days
12. Total Days 180 days
## Standard “N”: Number and Quantity

**Big Ideas:** In this course, students will be expected to take properties of familiar number systems and apply, compare, and contrast their application in new number systems. In particular, rational exponents of real numbers and the complex number system will be introduced and studied in detail. The reasonableness of an answer in the context of a problem will also be emphasized.

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<tr>
<th>Essential Questions</th>
<th>Enduring Understandings</th>
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<tbody>
<tr>
<td>What provocative questions will foster inquiry, understanding, and transfer of learning?</td>
<td>What will students understand about the big ideas?</td>
</tr>
</tbody>
</table>

- What are the important properties of exponents? Why does addition of exponents really represent multiplication?
- Do exponent properties apply to rational exponents?
- Can you find \( \sqrt{-1} \)?
- Can you take powers of complex numbers?
- Do the properties of real numbers apply to complex numbers?

Students will understand that…

- Integer exponents represent repeated multiplication.
- Rational exponents represent applying a root and a power to the base.
- Throughout our history, numbers have been created to represent quantities that may not be tangible. In advanced mathematics, we let \( i = \sqrt{-1} \).
- Addition, subtraction, and multiplication of complex numbers are defined over the commutative, associative, and distributive properties.

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<tr>
<th>Areas of Focus: Proficiencies (Cumulative Progress Indicators)</th>
<th>Examples, Outcomes, Assessments</th>
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<tbody>
<tr>
<td>Students will:</td>
<td>Instructional Focus:</td>
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<tr>
<td>(N-RN) explain how the definition of the meaning of rational exponents follows from extending the properties of integer exponents to those values, allowing for a notation for radicals in terms of rational exponents.</td>
<td>1. Properties of Exponents</td>
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<tr>
<td>(N-RN) rewrite expressions involving radicals and rational exponents using the properties of exponents.</td>
<td>2. Nth Roots and Rational Exponents</td>
</tr>
<tr>
<td>(N-Q) use units as a way to understand problems and to guide the solution of multi-step problems; chose and interpret units consistently in formulas; chose and interpret the scale and the origin in</td>
<td>3. Properties of Roots of Real Numbers</td>
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<td>4. Complex Numbers</td>
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<td>5. Solving any Quadratic Equation</td>
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<td>Sample Assessments:</td>
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<tr>
<td></td>
<td>a. Evaluate ( \sqrt[3]{-32} )</td>
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<tr>
<td></td>
<td>b. Simplify ( (5)^{1/4} )</td>
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<tr>
<td></td>
<td>c. Simplify ( 4\sqrt{3} + \sqrt{729} )</td>
</tr>
<tr>
<td></td>
<td>d. Simplify ( (2+7i) - (4 - 5i) )</td>
</tr>
<tr>
<td></td>
<td>( i^{42} )</td>
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<tr>
<td>e.</td>
<td>Solve ( 5x^2 - 2x + 1 = 0 ) over the complex numbers</td>
</tr>
<tr>
<td>f.</td>
<td>Factor ( 4x^2 + 100 ) over the complex numbers</td>
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</table>

**Projects**

- Students will be asked to research, discuss, and present the history of number systems. The development of the number 0 and the applications of \( i \) will be emphasized.

- Students will form small groups. Each group will be assigned one of the six properties of roots of real numbers:

  - \( a^m \cdot a^n = a^{m+n} \)
  - \( (a^m)^n = a^{mn} \)
  - \( (a \cdot b)^n = a^m \cdot b^n \)
  - \( a^m \) \( \frac{a^m}{a^n} = a^{m-n} \)
  - \( \frac{a^m}{b^n} \)
  - \( a^{-m} = \frac{1}{a^m} \)

Each group will be expected to make a short presentation (as a review) of their property. The presentation should include:

- an intuitive explanation of the property, ideally using a simple example and repeated integers
- an example involving rational exponents and/or roots
- an example that a group member struggled with, and how that error can be avoided
- an example for the rest of the class to try on their own

\( (N-Q) \) define appropriate quantities for the purpose of descriptive modeling.

\( (N-CN) \) know there is a complex number \( i \) such that \( i^2 = -1 \), and every complex number has the form \( a+bi \) with \( a \) and \( b \) real.

\( (N-CN) \) use the relation \( i^2 = -1 \) and the commutative, associative, and distributive properties to add, subtract, and multiply complex numbers.

\( (N-CN) \) find the conjugate of a complex number.

\( (N-CN) \) solve quadratic equations with real coefficients that have complex solutions.

\( (N-CN) \) extend polynomial identities (such as difference of squares) to complex numbers.

\( (N-CN) \) know the Fundamental Theorem of Algebra; show it is true for quadratic polynomials.
Instructional Strategies: Interdisciplinary Connections

Students will learn that complex numbers are widely used in the fields of physics and engineering, as well as in advanced mathematics. Students will learn the history of complex numbers, as well as the connections between complex numbers and fractal geometry, which appears in both art and nature.

Technology Integration

- Students will be encouraged to use calculators and other technology to validate solutions found using “pencil and paper”, particularly when manipulating expressions with rational exponents.


    The above link is a good resource for students to use when simplifying rational expressions.

  - [http://mathforum.org/johnandbetty/frame.htm](http://mathforum.org/johnandbetty/frame.htm)

    The above link is a “story book” that explains how numbers can be created to represent quantities that are not necessarily tangible. Applications of complex numbers are then described.

  - [http://www.ddewey.net/mandelbrot/](http://www.ddewey.net/mandelbrot/)

    The above link makes a connection between complex numbers and fractal geometry. The Mandelbrot Set is briefly illustrated.
### Media Literacy Integration

Students will use rational exponents to model exponential growth and decay. Students will be asked to find examples of growth and decay in the media.

### Global Perspectives

Students will investigate the simultaneous development of number systems in different cultures and regions.

The following skills and themes listed to the right should be reflected in the design of units and lessons for this course or content area.

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<tr>
<th>21st Century Skills:</th>
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<tbody>
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<td>Creativity and Innovation</td>
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<tr>
<td>Critical Thinking and Problem Solving</td>
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<td>Communication and Collaboration</td>
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<td>Information Literacy</td>
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<td>Media Literacy</td>
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<td>Life and Career Skills</td>
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<tr>
<th>21st Century Themes (as applies to content area):</th>
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<tr>
<td>Financial, Economic, Business, and Entrepreneurial Literacy</td>
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<tr>
<td>Civic Literacy</td>
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<td>Health Literacy</td>
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## Standard “A”: Algebra

### Big Ideas:
Students’ skills in applying algebraic properties to writing expressions, rearranging expressions, and solving equations will be extended to a wide variety of algebraic functions. Algebraic manipulations involving polynomial, rational, exponential, logarithmic, and trigonometric expressions and equations will be emphasized in this course. Students will be expected to convert tabular, written, or graphed mathematic relationships to algebraic models.

### Essential Questions

**What provocative questions will foster inquiry, understanding, and transfer of learning?**

- How can algebra be used to write expressions in more helpful forms?

- How can an equation’s solution(s) be interpreted on a graph?

- What are the similarities between different methods of equation solving?

- How can real-world problems be modeled with equations, systems of equations, or inequalities?

### Enduring Understandings

**What will students understand about the big ideas?**

Students will understand that…

- factoring and exponent properties can transform expressions into more useful forms.

- Solution(s) of an equation that has an expression equal to zero can be represented by the x-intercepts of the expressions’ graph.

- Solution(s) of an equation with two equal expressions can be interpreted as the points of intersection of the graphs of the two expressions.

- all equations that can be solved algebraically require inverse operations. Logarithms and exponential expressions, for example, are inverses and are used in solving equations.

- Solving equations algebraically can help to uncover points of intersection in systems, zeros of functions, or the location of specific function values.

### Areas of Focus: Proficiencies (Cumulative Progress Indicators)

**Examples, Outcomes, Assessments**

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<th>Instructional Focus:</th>
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<td>1. Literal Equations and Formulas</td>
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<td>2. Writing Equations of Lines</td>
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<td>3. Graphing Linear Inequalities in Two Variables</td>
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<td>4. Graphs of Absolute Value Equations</td>
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<table>
<thead>
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<th>Students will:</th>
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<tbody>
<tr>
<td>(A-SSE) use the structure of an expression to identify ways to rewrite it.</td>
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<tr>
<td>(A-SSE) factor a quadratic expression to reveal the zeros of the function it defines.</td>
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<tr>
<td>(A-SSE) complete the square in a quadratic</td>
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</tbody>
</table>
equation to reveal the maximum or minimum value of the function it defines.

(A-SSE) use the properties of exponents to transform expressions for exponential functions.

(A-APR) understand that polynomials form a system analogous to the integers, namely, that they are closed under addition, subtraction, and multiplication; add, subtract, and multiply polynomials.

(A-APR) know and apply the Remainder Theorem.

(A-APR) identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial.

(A-APR) rewrite simple rational expressions in different forms using inspection, long division, or, for the more complicated expressions, a computer algebra system.

(A-CED) create equations and inequalities (linear, quadratic, rational, exponential) in one variable and use them to solve problems.

(A-CED) create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.

(A-CED) represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or non-viable options in modeling contexts.

(A-CED) rearrange formulas to highlight a quantity of interest, using the same reasoning as solving equations.

(A-REI) solve simple rational and radical equations in one variable, and give examples of how extraneous solutions may arise.

(A-REI) use the method of completing the square to transform any quadratic equation in x into an equation of the form \((x - p)^2 = q\) that has the same solutions. Derive the quadratic formula from this form.

(A-REI) solve quadratic equations by inspection, taking square roots, completing the square, the quadratic formula, and factoring, as appropriate to the initial form of the equation.

(A-REI) prove that, given a system of two...
equations in two variables, replacing one
equation by the sum of that equation and a
multiple of the other produces a system with
the same solutions.

(A-REI) solve systems of linear equations
exactly and approximately.

(A-REI) solve a simple system consisting of
a linear equation and a quadratic equation in
two variables algebraically and graphically.

(A-REI) explain why the x-coordinates of
the points where the graphs of the equations
y=f(x) and y=g(x) intersect are the solutions
of the equation f(x)=g(x); find the solutions
approximately using technology; include
cases where f and g are linear, polynomial,
rational, absolute value, exponential,
logarithmic, and trigonometric.

(A-REI) graph the solutions to a linear
inequality in two variables as a half-plane,
and graph the solution set to a system of
linear inequalities in two variables as the
intersection of the corresponding half-
planes.

Ex2: A ball is thrown upward from the
ground with an initial velocity of 12
feet/second. Use the free-fall model to
determine how long it will stay in the air, and
what the maximum height of the ball will be.

Projects
• Grocery Receipts project: Use
grocery receipts to determine prices
of individual items.

• Using the determinant to solve
systems of linear equations.

• Passaic River Conservation project

• The Ultimate Box

• The Pendulum Swings

• Students will be asked to create a
system of inequalities that models a
scenario similar to the following:

You are the assistant manager of an
appliance store. You have two types
of stereo systems that you can order:
Type A costs $300 each with a profit
of $40 per unit, while Type B costs
$400 each with a profit of $60 per
unit. You expect a profit of at least
$4800 and you expect to sell at least
100 units. How many of each model
should you order to maximize profit?

Students will then graph the system
of equations, find the vertices of the
system algebraically, and optimize the
optimum quantity using the
procedure outlined by linear
programming.

• Students will model exponential
decay using M&M candies. Pouring
the candies on a plate and counting
the remaining candies will generate

\[ 4 \sin^2 x - 2 = 0 \]
data. Those that are “M&M” up will “decay”, and be removed. The process will continue until only a few candies remain. With their data, students will:
- use the TI-83 to generate a regression equation for pours vs. candies.
- if the process was reversed, and candies were added, how many pours would it take to generate 1,000,000,000 candies?
- graph the data and the regression equation together, and describe any patterns or deviations
- research a real-world example of exponential decay, and present their findings.

• Students will take any book and measure its height in inches. Students will then describe the height of a pile of such books, if the pile were to double each day. Students will be expected to:
  - write the pile’s height as a function of time in days
  - solve for the time at which the pile will reach: the ceiling (10 feet), the Empire State Building (1,250 feet), the moon (238,857 miles).
  - Compare results to students with books of different heights

Instructional Strategies:
  Interdisciplinary Connections

Students will be solving equations that are mathematical models for scenarios in the social sciences, physics, biology, and economics.

Technology Integration

• Technology (TI-83 graphing calculators, Geogebra, Mathematica software, and various web applets) will be used extensively to find solutions to equations that are too
complex to be solved by hand.

- [http://www.geogebra.org](http://www.geogebra.org)
  
  The above website allows students to graph equations, find points of intersection, and perform numerous other algebraic and geometric manipulations.

  
  The above website will solve any equation entered, provide a graphic interpretation of the solution, and perform many other mathematical tasks.

Media Literacy

Students will be asked to use resources online and in other textbooks and publications to find alternative ways to solve equations. Students will be asked to analyze these resources in terms of their usefulness and ease. These resources can be compiled into an online catalogue for future classes to use and add to.

Global Perspectives

Real data that describes different nations' economies and populations will be used as examples of linear growth, exponential growth, exponential decay, and periodic functions.

The following skills and themes listed to the right should be reflected in the design of units and lessons for this course or content area.

21st Century Skills:
- Creativity and Innovation
- Critical Thinking and Problem Solving
- Communication and Collaboration
- Information Literacy
- Media Literacy
- Life and Career Skills
### 21st Century Themes (as applies to content area):
- Financial, Economic, Business, and Entrepreneurial Literacy
- Civic Literacy
- Health Literacy

## Standard “F”: Functions

**Big Ideas:** Students will understand the concept of a function and be able to use function notation properly. A variety of “parent functions”, including power, polynomial, rational, exponential, logarithmic, and trigonometric functions, will represent different relationships between quantities. Students will be expected to create and interpret functions that are represented algebraically, numerically, graphically, and in written form.

### Essential Questions

**What provocative questions will foster inquiry, understanding, and transfer of learning?**

- What is a function?
- What are the benefits, and potential pitfalls, of using function notation?
- How do parent functions and transformations allow us to easily identify important properties of more complicated functions?

### Enduring Understandings

**What will students understand about the big ideas?**

- Students will understand that…
- a function is a type of relation that pairs an “input” value to exactly one “output” value
- function notation clearly denotes the name of the function, the rule assigned to the function, and the input into the function
- parent functions are (algebraically) the most basic of a certain type of function. Transformations of parent functions can be easily identified using function notation. For example, \( f(x - 2) + 3 \) indicates that the function \( f \) has been translated two units right and three units up.
- How can functions be created that model real-world data?

- Different relationships between sets of numbers are modeled by different functions. For example, quantities that change at a constant rate can be modeled by a linear function, while functions that oscillate can be modeled by a trigonometric function.

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<tbody>
<tr>
<td>Students will:</td>
<td>Instructional Focus:</td>
</tr>
<tr>
<td>(F-IF) understand that a function from one set to another set assigns each element of the domain to exactly one element of the range. If ( f ) is a function and ( x ) is an element of its domain, then ( f(x) ) denotes the output of ( f ) corresponding to the input ( x ). The graph of ( f ) is the graph of the equation ( y=f(x) ).</td>
<td>1. Quick Graphs of Linear Equations</td>
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<tr>
<td>(F-IF) use function notation, evaluate functions for inputs on their domains, and interpret statements that use function notation in terms of a context.</td>
<td>2. Graphs of Absolute Value Equations</td>
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<td>(F-IF) recognize that sequences are functions, sometimes defined recursively, whose domain is the subset of the integers.</td>
<td>3. Graphs of Quadratic Equations</td>
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<td>(F-IF) interpret key features of functions including: intercepts, intervals where the function is increasing, decreasing, positive, or negative, relative maximum and minimum values, end behavior, and periodicity.</td>
<td>4. Relations and Functions</td>
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<tr>
<td>(F-IF) relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes.</td>
<td>5. Function Operations</td>
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<tr>
<td>(F-IF) graph linear and quadratic functions and show intercepts, maxima, and minima.</td>
<td>6. Inverse Functions</td>
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<tr>
<td>(F-IF) graph square root, cube root, and piecewise-defined functions and absolute value functions.</td>
<td>7. Special Functions [Piecewise]</td>
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<tr>
<td>(F-IF) graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior.</td>
<td>8. Transformations of Graphs of Functions</td>
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<tr>
<td>(F-IF) graph rational functions, identifying zeros and asymptotes when suitable factorizations are available, and showing end behavior.</td>
<td>9. Recursive Functions and Finite Differences</td>
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<tr>
<td>(F-IF) graph exponential and logarithmic functions, showing intercepts and end</td>
<td>10. Compound Interest and Exponential Growth</td>
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<td>11. Graphing Square Root and Cube Root Functions</td>
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<td>12. Exponential Functions</td>
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<td>13. Logarithmic Functions</td>
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<td>14. Properties of Logarithms</td>
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<td>15. The Natural Base e</td>
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<td>16. Natural Logarithms</td>
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<td>17. Graphs of Polynomial Functions</td>
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<td>18. Graphs of Rational Functions</td>
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<td></td>
<td>19. Parabolas</td>
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<td>20. Circles</td>
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<td>21. Ellipses</td>
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<td>22. Hyperbolas</td>
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<td>23. Translations of Conics</td>
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<td>24. Right Triangle Trigonometry</td>
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<td>25. Angles of Rotation and Radian Measure</td>
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<td>26. Evaluating Trigonometric Functions</td>
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<td>27. Inverse Trigonometric Functions</td>
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<td>28. The Law of Sines</td>
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<td>29. The Law of Cosines</td>
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<td></td>
<td>30. Graphs of Sine and Cosine</td>
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</tbody>
</table>
behavior, and trigonometric functions, showing period, midline, and amplitude.
(F-I) use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context.
(F-I) use the properties of exponents to interpret expressions for exponential functions.
(F-BF) determine an explicit expression, a recursive process, or steps for calculation from a context.
(F-BF) compose functions.
(F-BF) write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the forms.
(F-BF) identify the effect on a graph of replacing f(x) by f(x) + k, kf(x), f(kx), and f(x + k) for specific values of k; find the value of k from a given graph; experiment with values of k using technology.
(F-BF) find inverse functions.
(F-BF) verify by composition that one function is the inverse of another.
(F-BF) read values of an inverse function from a graph or table, given that the function has an inverse.
(F-BF) read values of an inverse function from a graph or table, given that the function has an inverse.
(F-BF) produce an invertible function from a non-invertible function by restricting the domain.
(F-BF) understand the inverse relationship between exponents and logarithms and use this relationship to solve problems involving exponents and logarithms.
(F-LE) prove that linear functions grow by equal differences over equal intervals, and that exponential functions grow by equal factors over equal intervals.
(F-LE) recognize situations in which one quantity changes at a constant rate per unit interval relative to another.

Functions
31. Translations and Reflections of Graphs of Trig Functions
32. Trigonometric Identities
33. Sum and Difference Formulas
34. Double and Half Angle Formulas

Sample Assessments:
Ex1: Graph each, and include critical properties of each graph.

\[
f(x) = -2(x - 1)^2 + 4
\]
\[
f(x) = 3e^x
\]
\[
f(x) = \log_{10}(x - 1)
\]
\[
f(x) = \frac{x + 7}{2 - x}
\]
\[
f(x) = 3\sin(2x - \pi)
\]

Ex2: The table of data below represents an account’s balance after x years. Determine if the relationship is linear, exponential, or neither. If it is linear or exponential, write an explicit and recursive equation for the function.

<table>
<thead>
<tr>
<th>X (years)</th>
<th>B (Account balance)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>$4,250.00</td>
</tr>
<tr>
<td>1</td>
<td>$4,403.00</td>
</tr>
<tr>
<td>2</td>
<td>$4,561.51</td>
</tr>
</tbody>
</table>

How long will it take the account to reach
(F-LE) recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another.

(F-LE) construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs.

(F-LE) observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or as a polynomial function.

(F-TF) understand radian measure of an angle is the length of the arc on the unit circle subtended by the angle.

(F-TF) explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as the radian measures of angles traversed counterclockwise around the unit circle.

(F-TF) use special triangles to determine geometrically the values of sine, cosine, and tangent for $\frac{\pi}{3}$, $\frac{\pi}{4}$, and $\frac{\pi}{6}$.

(F-TF) use the unit circle to explain symmetry (odd and even) and periodicity of trigonometric functions.

(F-TF) choose trigonometric functions to model periodic phenomena with specified amplitudes, frequency, and midline.

(F-TF) understand that restricting a trigonometric function to a domain on which it is always increasing or always decreasing allows its inverse to be constructed.

(F-TF) use inverse functions to solve trigonometric equations that arise in modeling contexts; evaluate the solutions using technology, and interpret them in terms of the context.

(F-TF) prove the Pythagorean Identity $\sin^2 x + \cos^2 x = 1$ and use it to find $\sin(x)$, $\cos(x)$, or $\tan(x)$ given one of these and a quadrant.

(F-TF) prove the addition and subtraction formulas for sine, cosine, and tangent and

Projects
- Hang Man, Swing Man, Tarzan
- Sine City – Go Global!
- Graphing Toolkit Poster
- Story Problems From A Graph
- CBR and Motion Detector
- Golf Ball Lab: Your Personal Parabola!
- Harry “Conic” Jr. – Graphing Calculator Project
- Who Am I? Paper Folding
- You are the LUB of my life!
- The Ultimate Box: Students will be asked to create an open box out of cardboard by cutting out squares from each corner of the cardboard and folding up the resulting flaps.
  -Write polynomial functions to model the outside surface area and the volume of the resulting box.
  -Graph the function for the volume of the box over a reasonable domain.
  -Use technology to determine the maximum value of the volume function. What sized squares should be cut out to maximize volume?
- Students will be asked to research phenomena that occur periodically in nature (daily average temperature for a city over a given year, or time of high and low tides are recommended topics). Students will then:
  -find data for a particular city on the internet (http://www.weather.com)
  -graph the data points using Geogebra

$10,000?"
use them to solve problems. (http://www.geogebra.com) or Microsoft Excel
-identify (approximately): the range of the data, the period of the data, the amplitude of the data
-use the values found above to write two regression equations for the data (one must be a sine function, the other a cosine function)
-use the regression equations (and algebraic techniques) to make predictions.

Instructional Strategies:
   Interdisciplinary Connections

Students will be writing and analyzing functions that are mathematical models for scenarios in the social sciences, physics, biology, and economics. Growth and decay in biology and finance will be emphasized.

Technology Integration

- Technology (TI-83 graphing calculators, Geogebra, Mathematica software, and various web applets) will be used extensively to graph and analyze functions that cannot be easily graphed by hand, or that do not have a parent function discussed in the course.


   The above website shows the relationship between the unit circle and the numeric values of the six trigonometric functions at different angles.


   The above website allows students to explore transformations of trigonometric functions by changing
the equation’s parameters.

* [http://math.hws.edu/javamath/basic_applets/SliderGraph.html](http://math.hws.edu/javamath/basic_applets/SliderGraph.html)

The above website allows users to enter a function, and then observe transformations using sliders.

Media Literacy

Students will be asked to find examples in the media of exponential growth and decay. Students will be expected to find websites, newspaper articles, video clips, and other acceptable media that discuss a real-world phenomenon or current event that is an example of exponential behavior.

Global Perspectives

Real data that describes different nations’ economies and populations will be used as examples of linear growth, exponential growth, exponential decay, and periodic functions.

The following skills and themes listed to the right should be reflected in the design of units and lessons for this course or content area.

<table>
<thead>
<tr>
<th>21st Century Skills:</th>
</tr>
</thead>
<tbody>
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<td>Creativity and Innovation</td>
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<td>Financial, Economic, Business, and Entrepreneurial Literacy</td>
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<td>Civic Literacy</td>
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<tr>
<td>Health Literacy</td>
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</table>
## Standard “S”: Statistics and Probability

### Big Ideas:
Students will be exposed to concepts in statistics including how to summarize data (numerically and graphically) and how to interpret these summaries. Students will also be able to solve problems involve combinations, permutations, and probability.

### Enduring Understandings
What will students understand about the big ideas?

<table>
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<tr>
<th>Essential Questions</th>
<th>Enduring Understandings</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>What provocative questions will foster inquiry, understanding, and transfer of learning?</strong></td>
<td>Students will understand that…</td>
</tr>
<tr>
<td>- How can data be numerically summarized? How can these summaries tell different stories?</td>
<td>- the mean and median measure the center of a distribution, while the standard deviation and five-number summary measure the spread of the standard deviation.</td>
</tr>
<tr>
<td>- How can data be graphically summarized? How can these summaries tell different stories?</td>
<td>- histograms, box and whisker plots, and stem and leaf plots can visually represent the center and spread of a distribution in slightly different ways.</td>
</tr>
<tr>
<td>- How can combinations and permutations of objects be counted efficiently?</td>
<td>- Patterns exist that allow for the number of combinations and permutations of a set to be counted efficiently; although, for small sample spaces, systematic listing can be reasonable.</td>
</tr>
<tr>
<td>- What is random behavior? How can probability help to model random processes?</td>
<td>- The likelihood of random events occurring can be predicted using the laws of probability.</td>
</tr>
</tbody>
</table>

### Areas of Focus: Proficiencies
(Cumulative Progress Indicators)

<table>
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<th>Examples, Outcomes, Assessments</th>
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<tr>
<td><strong>Instructional Focus:</strong></td>
</tr>
<tr>
<td>1. Exploring Data: Tables and Graphs</td>
</tr>
<tr>
<td>2. Exploring Data: Fitting a Line to Data</td>
</tr>
<tr>
<td>3. Exploring Data: Measures of Central Tendency</td>
</tr>
<tr>
<td>4. Exploring Data: Measures of</td>
</tr>
<tr>
<td>Sample Assessments:</td>
</tr>
<tr>
<td>---------------------</td>
</tr>
<tr>
<td>Ex1: How many different license plates can be made if 7 characters must be used, any letter or number is allowed in any position, but no character can be used more than once?</td>
</tr>
<tr>
<td>Ex2: Using the data from the 2010 US Census [not shown here], create a histogram, box and whisker plot, and stem and leaf plot of each state’s total population. Use each graph to describe the center and spread of the distribution. How do these graphs display the same set of data differently?</td>
</tr>
<tr>
<td>Projects:</td>
</tr>
<tr>
<td>- Live and Let Dice Lucky “7’s”</td>
</tr>
<tr>
<td>- Phone Numbers, License Plates, and Cell Phones</td>
</tr>
<tr>
<td>- Students will create a simple statistical analysis of a set of data of their choice in the social sciences or physical sciences. The data should have at least 40 elements. Students will:</td>
</tr>
<tr>
<td>- Visually represent the data using a histogram, stem and leaf plot, and a box and whisker plot</td>
</tr>
<tr>
<td>- Numerically summarize the data by finding the mean and standard deviation, and the median and five-number summary.</td>
</tr>
<tr>
<td>- Summarize their findings from both the graphs and computations.</td>
</tr>
<tr>
<td>Students should compare and contrast their results, as well as describe which graphic and numeric...</td>
</tr>
</tbody>
</table>
summaries are most appropriate and why.

- Students will investigate the effect of outliers on a set of data. Students will take a set of data that is approximately “normally distributed” and create a histogram and numeric summary. Then, students will add a single extreme outlier, and repeat the computations. Students should present their results using a poster or Microsoft Powerpoint.

Instructional Strategies:
Interdisciplinary Connections

Students will explore how data and probability can be used to make decisions and predictions in the social and physical sciences.

Technology Integration

- Technology (TI-83 graphing calculators, Geogebra, Mathematica software, and various web applets) will be used to simulate random behavior, generate graphs of data, and quickly find measures of central tendency and spread.

- [http://www.keypress.com/x5656.xml](http://www.keypress.com/x5656.xml)

  Students can use Fathom software to easily generate graphs and numeric summaries of data.

- [http://www.stat.tamu.edu/~west/ph/meanmedian.html](http://www.stat.tamu.edu/~west/ph/meanmedian.html)

  The applet above demonstrates the effect that outliers can have on the mean and median. Students can use their own data and observe the results of modifying a single element.

Media Literacy
Students will be asked to scour media sources for examples of statistics in the news. Students will be encouraged to examine these studies for potential biases. Through this activity, the unit can be extended to briefly discuss sampling techniques.

**Global Perspectives**

Real data that compares the growth rates of different nations’ economies and populations will be used to motivate statistics topics.

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Texts and Resources:


TI-84+ Graphing Calculator. Texas Instruments.