Summit Public Schools  
Summit, New Jersey  

Grade Level / Content Area: 10-11 / Mathematics  
Length of Course: 1 Year  

Algebra 2/Trigonometry

Course Description:
The overall goals of the course are: to explore a variety of functions that can be used to model relationships between sets of numbers; to introduce the set of complex numbers; to build equation-solving skills; to introduce basic data analysis and probability. Students will be expected to work with relations that are in a variety of representations, including algebraic, tabular, graphic, and verbal forms. Real-world data will be used to motivate and extend all topics. The TI-83 graphing calculator and web-based technologies will be used extensively to assist in solving complicated problems. Students will be expected to communicate mathematics clearly in written, verbal, and algebraic forms.

Course Pacing:

1. Review of Basic Algebra / Linear Equations (Chapters 1 and 2) 12 days
2. Systems of Linear Equations and Inequalities (Chapter 3) 13 days
3. Quadratic Equations and Parabolas (Chapter 5) 16 days
4. Functions (Chapter 6) 14 days
5. Powers, Roots, and Radicals (Chapter 7) 12 days
6. Exponential and Logarithmic Functions (Chapter 8) 15 days
7. Probability and Statistics (Chapter 15) 12 days
8. Polynomials (Chapter 9) 16 days
9. Rational Functions (Chapter 10) 16 days
10. Trigonometric Ratios and Functions (Chapter 13) 17 days
11. Trigonometric Graphs, Identities, and Equations (Chapter 14) 16 days
12. Conic Sections (Chapter 11) 10 days
### Standard “N”: Number and Quantity

**Big Ideas:** In this course, students will be expected to take properties of familiar number systems and apply, compare, and contrast their application in new number systems. In particular, rational exponents of real numbers and the complex number system will be introduced and studied in detail. The reasonableness of an answer in the context of a problem will also be emphasized.

<table>
<thead>
<tr>
<th>Essential Questions</th>
<th>Enduring Understandings</th>
</tr>
</thead>
<tbody>
<tr>
<td><em>What provocative questions will foster inquiry, understanding, and transfer of learning?</em></td>
<td><em>What will students understand about the big ideas?</em></td>
</tr>
<tr>
<td>- What are the important properties of exponents? Why does addition of exponents really represent multiplication?</td>
<td>Students will understand that…</td>
</tr>
<tr>
<td>- Do exponent properties apply to rational exponents?</td>
<td>- Integer exponents represent repeated multiplication.</td>
</tr>
<tr>
<td>- Can you find $\sqrt{-1}$?</td>
<td>- Rational exponents represent applying a root and a power to the base.</td>
</tr>
<tr>
<td>- Do the properties of real numbers apply to complex numbers?</td>
<td>- Throughout our history, numbers have been created to represent quantities that may not be tangible. In advanced mathematics, we let $i = \sqrt{-1}$.</td>
</tr>
<tr>
<td>- Addition, subtraction, and multiplication of complex numbers are defined over the commutative, associative, and distributive properties.</td>
<td></td>
</tr>
</tbody>
</table>

### Areas of Focus: Proficiencies (Cumulative Progress Indicators)

**Students will:**

(N-RN) explain how the definition of the meaning of rational exponents follows from extending the properties of integer exponents to those values, allowing for a notation for radicals in terms of rational exponents.

(N-RN) rewrite expressions involving radicals and rational exponents using the properties of exponents.

(N-Q) use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays.

(N-Q) define appropriate quantities for the

### Examples, Outcomes, Assessments

**Instructional Focus:**

1. Properties of Exponents
2. Nth Roots and Rational Exponents
3. Properties of Roots of Real Numbers
4. Complex Numbers
5. Solving any Quadratic Equation

**Sample Assessments:**

a. Evaluate $\sqrt[5]{-32}$

b. Simplify $(5^{1/4}(5)^{-3/4}$

c. Simplify $4\sqrt{3} + \sqrt[3]{729}$

d. Simplify
   i. $(2+7i) - (4 - 5i)$
   ii. $i^{42}$
<table>
<thead>
<tr>
<th>Purpose of Descriptive Modeling.</th>
<th>( N ) - ( CN )</th>
<th>( \text{Know there is a complex number } i \text{ such that } i^2 = -1 ), and every complex number has the form ( a + bi ) with ( a ) and ( b ) real.</th>
</tr>
</thead>
<tbody>
<tr>
<td>( N ) - ( CN )</td>
<td>use the relation ( i^2 = -1 ) and the commutative, associative, and distributive properties to add, subtract, and multiply complex numbers.</td>
<td></td>
</tr>
<tr>
<td>( N ) - ( CN )</td>
<td>find the conjugate of a complex number.</td>
<td></td>
</tr>
<tr>
<td>( N ) - ( CN )</td>
<td>solve quadratic equations with real coefficients that have complex solutions.</td>
<td></td>
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<tr>
<td>( N ) - ( CN )</td>
<td>extend polynomial identities (such as difference of squares) to complex numbers.</td>
<td></td>
</tr>
<tr>
<td>( N ) - ( CN )</td>
<td>know the Fundamental Theorem of Algebra; show it is true for quadratic polynomials.</td>
<td></td>
</tr>
</tbody>
</table>

Projects

- Students will be asked to research, discuss, and present the history of number systems. The development of the number 0 and the applications of \( i \) will be emphasized.
- Students will form small groups. Each group will be assigned one of the six properties of roots of real numbers:
  
  \[
  a^m \cdot a^n = a^{m+n} \\
  \left( a^m \right)^n = a^{mn} \\
  (a \cdot b)^m = a^m \cdot b^m \\
  a^n \div a^m = a^{n-m} \\
  \left( \frac{a}{b} \right)^m = \frac{a^m}{b^m} \\
  a^{-m} = \frac{1}{a^m}
  \]

  Each group will be expected to make a short presentation (as a review) of their property. The presentation should include:
  - an intuitive explanation of the property, ideally using a simple example and repeated integers
  - an example involving rational exponents and/or roots
  - an example that a group member struggled with, and how that error can be avoided
  - an example for the rest of the class to try on their own
Instructional Strategies:
  Interdisciplinary Connections

Students will learn that complex numbers are widely used in the fields of physics and engineering, as well as in advanced mathematics. Students will learn the history of complex numbers, as well as the connections between complex numbers and fractal geometry, which appears in both art and nature.

Technology Integration

• Students will be encouraged to use calculators and other technology to validate solutions found using “pencil and paper”, particularly when manipulating expressions with rational exponents.


  The above link is a good resource for students to use when simplifying rational expressions.

• [http://mathforum.org/johnandbetty/frame.htm](http://mathforum.org/johnandbetty/frame.htm)

  The above link is a “story book” that explains how numbers can be created to represent quantities that are not necessarily tangible. Applications of complex numbers are then described.

• [http://www.ddewey.net/mandelbrot/](http://www.ddewey.net/mandelbrot/)

  The above link makes a connection between complex numbers and fractal geometry. The Mandelbrot Set is briefly illustrated.
<table>
<thead>
<tr>
<th>Media Literacy Integration</th>
</tr>
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<tbody>
<tr>
<td>Students will use rational exponents to model exponential growth and decay. Students will be asked to find examples of growth and decay in the media.</td>
</tr>
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<th>Global Perspectives</th>
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<td>Students will investigate the simultaneous development of number systems in different cultures and regions.</td>
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The following skills and themes listed to the right should be reflected in the design of units and lessons for this course or content area.

<table>
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<th>21st Century Skills:</th>
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<th>21st Century Themes (as applies to content area):</th>
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<tr>
<td>Financial, Economic, Business, and Entrepreneurial Literacy</td>
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<tr>
<td>Civic Literacy</td>
</tr>
<tr>
<td>Health Literacy</td>
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Standard “A”: Algebra

**Big Ideas**: Students’ skills in applying algebraic properties to writing expressions, rearranging expressions, and solving equations will be extended to a wide variety of algebraic functions. Algebraic manipulations involving polynomial, rational, exponential, logarithmic, and trigonometric expressions and equations will be emphasized in this course. Students will be expected to convert tabular, written, or graphed mathematic relationships to algebraic models.

**Essential Questions**
*What provocative questions will foster inquiry, understanding, and transfer of learning?*

- How can algebra be used to write expressions in more helpful forms?
- How can an equation’s solution(s) be interpreted on a graph?
- What are the similarities between different methods of equation solving?
- How can real-world problems be modeled with equations, systems of equations, or inequalities?

**Enduring Understandings**
*What will students understand about the big ideas?*

Students will understand that…

- Factoring and exponent properties can transform expressions into more useful forms.
- Solution(s) of an equation that has an expression equal to zero can be represented by the x-intercepts of the expressions’ graph.
- Solution(s) of an equation with two equal expressions can be interpreted as the points of intersection of the graphs of the two expressions.
- All equations that can be solved algebraically require inverse operations. Logarithms and exponential expressions, for example, are inverses and are used in solving equations.
- Solving equations algebraically can help to uncover points of intersection in systems, zeros of functions, or the location of specific function values.

**Areas of Focus: Proficiencies**
*Cumulative Progress Indicators*

<table>
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<tr>
<th>Students will:</th>
<th>Examples, Outcomes, Assessments</th>
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<tbody>
<tr>
<td>(A-SSE) use the structure of an expression to identify ways to rewrite it.</td>
<td>Instructional Focus:</td>
</tr>
<tr>
<td>(A-SSE) factor a quadratic expression to reveal the zeros of the function it defines.</td>
<td>1. Literal Equations and Formulas</td>
</tr>
<tr>
<td>(A-SSE) complete the square in a quadratic</td>
<td>2. Writing Equations of Lines</td>
</tr>
<tr>
<td></td>
<td>3. Graphing Linear Inequalities in Two Variables</td>
</tr>
<tr>
<td></td>
<td>4. Graphs of Absolute Value Equations</td>
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equation to reveal the maximum or minimum value of the function it defines.

(A-SSE) use the properties of exponents to transform expressions for exponential functions.

(A-APR) understand that polynomials form a system analogous to the integers, namely, that they are closed under addition, subtraction, and multiplication; add, subtract, and multiply polynomials.

(A-APR) know and apply the Remainder Theorem.

(A-APR) identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial.

(A-APR) rewrite simple rational expressions in different forms using inspection, long division, or, for the more complicated expressions, a computer algebra system.

(A-CED) create equations and inequalities (linear, quadratic, rational, exponential) in one variable and use them to solve problems.

(A-CED) create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.

(A-CED) represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or non-viable options in modeling contexts.

(A-CED) rearrange formulas to highlight a quantity of interest, using the same reasoning as solving equations.

(A-REI) solve simple rational and radical equations in one variable, and give examples of how extraneous solutions may arise.

(A-REI) use the method of completing the square to transform any quadratic equation in x into an equation of the form $(x - p)^2 = q$ that has the same solutions. Derive the quadratic formula from this form.

(A-REI) solve quadratic equations by inspection, taking square roots, completing the square, the quadratic formula, and factoring, as appropriate to the initial form of the equation.

(A-REI) prove that, given a system of two

<table>
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<tr>
<td><strong>Ex1:</strong> Solve for $x$</td>
</tr>
<tr>
<td>a. $a = x - bx$</td>
</tr>
<tr>
<td>b. $</td>
</tr>
<tr>
<td>c. $\begin{cases} x - 3y = -3 \ 2x + y = 8 \end{cases}$</td>
</tr>
<tr>
<td>d. $16x + 6x^2 + 5 = 4$</td>
</tr>
<tr>
<td>e. $\sqrt{2x + 10} - 2\sqrt{x} = 0$</td>
</tr>
<tr>
<td>f. $3e^{5x} = 8$</td>
</tr>
<tr>
<td>g. $-5 + 2\ln 3x = 5$</td>
</tr>
<tr>
<td>h. $x^4 + x^3 - x^2 + x - 2 = 0$</td>
</tr>
<tr>
<td>i. $x + \frac{2}{x} = \frac{5}{x + 2}$</td>
</tr>
<tr>
<td>j. $4 \sin^2 x - 2 = 0$</td>
</tr>
</tbody>
</table>
equations in two variables, replacing one equation by the sum of that equation and a multiple of the other produces a system with the same solutions.

(A-REI) solve systems of linear equations exactly and approximately.

(A-REI) solve a simple system consisting of a linear equation and a quadratic equation in two variables algebraically and graphically.

(A-REI) explain why the x-coordinates of the points where the graphs of the equations \( y=f(x) \) and \( y=g(x) \) intersect are the solutions of the equation \( f(x)=g(x) \); find the solutions approximately using technology; include cases where \( f \) and \( g \) are linear, polynomial, rational, absolute value, exponential, logarithmic, and trigonometric.

(A-REI) graph the solutions to a linear inequality in two variables as a half-plane, and graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes.

Ex2: A ball is thrown upward from the ground with an initial velocity of 12 feet/second. Use the free-fall model to determine how long it will stay in the air, and what the maximum height of the ball will be.

Projects

• Students will be asked to create a system of inequalities that models a scenario similar to the following:

You are the assistant manager of an appliance store. You have two types of stereo systems that you can order: Type A costs $300 each with a profit of $40 per unit, while Type B costs $400 each with a profit of $60 per unit. You expect a profit of at least $4800 and you expect to sell at least 100 units. How many of each model should you order to maximize profit?

Students will then graph the system of equations, find the vertices of the system algebraically, and optimize the optimum quantity using the procedure outlined by linear programming.

• Students will model exponential decay using M&M candies. Data will be generated by pouring the candies on a plate and counting the remaining candies. Those that are “M&M” up will “decay”, and be removed. The process will continue until only a few candies remain. With their data, students will:
  - use the TI-83 to generate a regression equation for pours vs. candies.
  - if the process were reversed, and candies were added, how many pours would it take to generate 1,000,000,000 candies?
  - graph the data and the regression equation together, and describe any patterns or deviations
  - research a real-world example of
exponential decay, and present their findings.

- Students will take any book and measure its height in inches. Students will then describe the height of a pile of such books, if the pile were to double each day. Students will be expected to:
  - write the pile’s height as a function of time in days
  - solve for the time at which the pile will reach: the ceiling (10 feet), the Empire State Building (1,250 feet), the moon (238,857 miles).
  - Compare results to students with books of different heights

Instructional Strategies:
  Interdisciplinary Connections

Students will be solving equations that are mathematical models for scenarios in the social sciences, physics, biology, and economics.

Technology Integration

- Technology (TI-83 graphing calculators, Geogebra, Mathematica software, and various web applets) will be used extensively to find solutions to equations that are too complex to be solved by hand.

  - [http://www.geogebra.org](http://www.geogebra.org)
    The above website allows students to graph equations, find points of intersection, and perform numerous other algebraic and geometric manipulations.

    The above website will solve any equation entered, provide a graphic interpretation of the solution, and perform many other mathematical
Media Literacy

Students will be asked to use resources online and in other textbooks and publications to find alternative ways to solve equations. Students will be asked to analyze these resources in terms of their usefulness and ease. These resources can be compiled into an online catalogue for future classes to use and add to.

Global Perspectives

Real data that describes different nations’ economies and populations will be used as examples of linear growth, exponential growth, exponential decay, and periodic functions.

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Creativity and Innovation  
Critical Thinking and Problem Solving  
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Information Literacy  
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Life and Career Skills  
21st Century Themes (as applies to content area):  
Financial, Economic, Business, and Entrepreneurial Literacy  
Civic Literacy  
Health Literacy |
## Standard “F”: Functions

### Big Ideas:
Students will understand the concept of a function and be able to use function notation properly. A variety of “parent functions”, including power, polynomial, rational, exponential, logarithmic, and trigonometric functions, will represent different relationships between quantities. Students will be expected to create and interpret functions that are represented algebraically, numerically, graphically, and in written form.

### Essential Questions
*What provocative questions will foster inquiry, understanding, and transfer of learning?*

- What is a function?
- What are the benefits, and potential pitfalls, of using function notation?
- How do parent functions and transformations allow us to easily identify important properties of more complicated functions?
- How can functions be created that model real-world data?

### Enduring Understandings
*What will students understand about the big ideas?*

- a function is a type of relation that pairs an “input” value to exactly one “output” value
- function notation clearly denotes the name of the function, the rule assigned to the function, and the input into the function
- parent functions are (algebraically) the most basic of a certain type of function. Transformations of parent functions can be easily identified using function notation. For example, \( f(x - 2) + 3 \) indicates that the function \( f \) has been translated two units right and three units up.
- Different relationships between sets of numbers are modeled by different functions. For example, quantities that change at a constant rate can be modeled by a linear function, while functions that oscillate can be modeled by a trigonometric function.

### Areas of Focus: Proficiencies (Cumulative Progress Indicators)

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<thead>
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<th>Examples, Outcomes, Assessments</th>
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<tbody>
<tr>
<td>Instructional Focus:</td>
</tr>
<tr>
<td>1. Quick Graphs of Linear Equations</td>
</tr>
<tr>
<td>2. Graphs of Absolute Value Equations</td>
</tr>
</tbody>
</table>
domain to exactly one element of the range.
If f is a function and x is an element of its
domain, then f(x) denotes the output of f
corresponding to the input x. The graph of f
is the graph of the equation y=f(x).

(F-IF) use function notation, evaluate
functions for inputs on their domains, and
interpret statements that use function
notation in terms of a context.

(F-IF) recognize that sequences are
functions, sometimes defined recursively,
whose domain is the subset of the integers.

(F-IF) interpret key features of functions
including: intercepts, intervals where the
function is increasing, decreasing, positive, or
negative, relative maximum and minimum
values, end behavior, and periodicity.

(F-IF) relate the domain of a function to its
graph and, where applicable, to the
quantitative relationship it describes.

(F-IF) graph linear and quadratic functions
and show intercepts, maxima, and minima.

(F-IF) graph square root, cube root, and
piecewise-defined functions and absolute
value functions.

(F-IF) graph polynomial functions,
identifying zeros when suitable factorizations
are available, and showing end behavior.

(F-IF) graph rational functions, identifying
zeros and asymptotes when suitable
factorizations are available, and showing end
behavior.

(F-IF) graph exponential and logarithmic
functions, showing intercepts and end
behavior, and trigonometric functions,
showing period, midline, and amplitude.

(F-IF) use the process of factoring and
completing the square in a quadratic function
to show zeros, extreme values, and symmetry
of the graph, and interpret these in terms of
a context.

(F-IF) use the properties of exponents to
interpret expressions for exponential
functions.

(F-BF) determine an explicit expression, a
recursive process, or steps for calculation
from a context.

(F-BF) compose functions.

(F-BF) write arithmetic and geometric
sequences both recursively and with an explicit formula, use them to model situations, and translate between the forms.

(F-BF) identify the effect on a graph of replacing \( f(x) \) by \( f(x)+k \), \( kf(x) \), \( f(kx) \), and \( f(x+k) \) for specific values of \( k \); find the value of \( k \) from a given graph; experiment with values of \( k \) using technology.

(F-BF) find inverse functions.

(F-BF) verify by composition that one function is the inverse of another.

(F-BF) read values of an inverse function from a graph or a table, given that the function has an inverse.

(F-BF) read values of an inverse function from a graph or table, given that the function has an inverse.

(F-BF) produce an invertible function from a non-invertible function by restricting the domain.

(F-BF) understand the inverse relationship between exponents and logarithms and use this relationship to solve problems involving exponents and logarithms.

(F-LE) prove that linear functions grow by equal differences over equal intervals, and that exponential functions grow by equal factors over equal intervals.

(F-LE) recognize situations in which one quantity changes at a constant rate per unit interval relative to another.

(F-LE) recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another.

(F-LE) construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs.

(F-LE) observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or as a polynomial function.

(F-TF) understand radian measure of an angle is the length of the arc on the unit circle subtended by the angle.

Sample Assessments:

Ex1: Graph each, and include critical properties of each graph.

\[
\begin{align*}
  f(x) &= -2(x - 1)^2 + 4 \\
  f(x) &= 3e^x \\
  f(x) &= \log_{10}(x - 1) \\
  f(x) &= \frac{x + 7}{2 - x} \\
  f(x) &= 3\sin(2x - \pi)
\end{align*}
\]

Ex2: The table of data below represents an account’s balance after \( x \) years. Determine if the relationship is linear, exponential, or neither. If it is linear or exponential, write an explicit and recursive equation for the function.

<table>
<thead>
<tr>
<th>( x ) (years)</th>
<th>( B ) (Account balance)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>$4,250.00</td>
</tr>
<tr>
<td>1</td>
<td>$4,403.00</td>
</tr>
<tr>
<td>2</td>
<td>$4,561.51</td>
</tr>
</tbody>
</table>

How long will it take the account to reach $10,000?

Projects

- Students will be asked to create an open box out of cardboard by cutting out squares from each corner of the cardboard and folding up the resulting flaps. Given that the cut out squares are \( x \)-by-\( x \) inches:
  - Write polynomial functions to model the outside surface area and the volume of the resulting box.
  - Graph the function for the volume of the box over a reasonable domain.
  - Use technology to determine the maximum value of the volume function. What sized squares should be cut out to maximize volume?
(F-TF) explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as the radian measures of angles traversed counterclockwise around the unit circle.

(F-TF) use special triangles to determine geometrically the values of sine, cosine, tangent for $\frac{\pi}{3}, \frac{\pi}{4}, \text{ and } \frac{\pi}{6}$.

(F-TF) use the unit circle to explain symmetry (odd and even) and periodicity of trigonometric functions.

(F-TF) chose trigonometric functions to model periodic phenomena with specified amplitudes, frequency, and midline.

(F-TF) understand that restricting a trigonometric function to a domain on which it is always increasing or always decreasing allows its inverse to be constructed.

(F-TF) use inverse functions to solve trigonometric equations that arise in modeling contexts; evaluate the solutions using technology, and interpret them in terms of the context.

(F-TF) prove the Pythagorean Identity $\sin^2 x + \cos^2 x = 1$ and use it to find $\sin(x)$, $\cos(x)$, or $\tan(x)$ given one of these and a quadrant.

| Instructional Strategies: | Students will be asked to research phenomena that occur periodically in nature (daily average temperature for a city over a given year, or time of high and low tides are recommended topics). Students will then:
- find data for a particular city on the internet (http://www.weather.com)
- graph the data points using Geogebra (http://www.geogebra.com) or Microsoft Excel
- identify (approximately): the range of the data, the period of the data, the amplitude of the data
- use the values found above to write two regression equations for the data (one must be a sine function, the other a cosine function)
- use the regression equations (and algebraic techniques) to make predictions. |

| Technology Integration | Students will be writing and analyzing functions that are mathematical models for scenarios in the social sciences, physics, biology, and economics. Growth and decay in biology and finance will be emphasized. |

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- use the regression equations (and algebraic techniques) to make predictions. |

| Technology Integration | Students will be writing and analyzing functions that are mathematical models for scenarios in the social sciences, physics, biology, and economics. Growth and decay in biology and finance will be emphasized. |

| Students will be asked to research phenomena that occur periodically in nature (daily average temperature for a city over a given year, or time of high and low tides are recommended topics). Students will then:
- find data for a particular city on the internet (http://www.weather.com)
- graph the data points using Geogebra (http://www.geogebra.com) or Microsoft Excel
- identify (approximately): the range of the data, the period of the data, the amplitude of the data
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The above website shows the relationship between the unit circle and the numeric values of the six trigonometric functions at different angles.


The above website allows students to explore transformations of trigonometric functions by changing the equation’s parameters.

- [http://math.hws.edu/javamath/basic_applets/SliderGraph.html](http://math.hws.edu/javamath/basic_applets/SliderGraph.html)

The above website allows users to enter a function, and then observe transformations using sliders.

### Media Literacy

Students will be asked to find examples in the media of exponential growth and decay. Students will be expected to find websites, newspaper articles, video clips, and other acceptable media that discuss a real-world phenomenon or current event that is an example of exponential behavior.

### Global Perspectives

Real data that describes different nations’ economies and populations will be used as examples of linear growth, exponential growth, exponential decay, and periodic functions.

<table>
<thead>
<tr>
<th>The following skills and themes listed to the right should be reflected in the design of units and lessons for this course or content area.</th>
<th>21st Century Skills:</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Creativity and Innovation</td>
</tr>
</tbody>
</table>
### Standard “S”: Statistics and Probability

**Big Ideas:** Students will be exposed to concepts in statistics including how to summarize data (numerically and graphically) and how to interpret these summaries. Students will also be able to solve problems involve combinations, permutations, and probability.

<table>
<thead>
<tr>
<th>Essential Questions</th>
<th>Enduring Understandings</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>What provocative questions will foster inquiry, understanding, and transfer of learning?</strong></td>
<td><strong>What will students understand about the big ideas?</strong></td>
</tr>
<tr>
<td>- How can data be numerically summarized? How can these summaries tell different stories?</td>
<td>- the mean and median measure the center of a distribution, while the standard deviation and five-number summary measure the spread of the standard deviation.</td>
</tr>
<tr>
<td>- How can data be graphically summarized? How can these summaries tell different stories?</td>
<td>- histograms, box and whisker plots, and stem and leaf plots can visually represent the center and spread of a distribution in slightly different ways.</td>
</tr>
<tr>
<td>- How can combinations and permutations of objects be counted efficiently?</td>
<td>- Patterns exist that allow for the number of combinations and permutations of a set to be counted efficiently; although, for small sample spaces, systematic listing can be reasonable.</td>
</tr>
</tbody>
</table>
- What is random behavior? How can probability help to model random processes?

- The likelihood of random events occurring can be predicted using the laws of probability.

<table>
<thead>
<tr>
<th>Areas of Focus: Proficiencies (Cumulative Progress Indicators)</th>
<th>Examples, Outcomes, Assessments</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Students will:</strong></td>
<td><strong>Instructional Focus:</strong></td>
</tr>
<tr>
<td>(S-ID) represent data with plots on the real number line (dot plots, histograms, and box plots).</td>
<td>1. Exploring Data: Tables and Graphs</td>
</tr>
<tr>
<td>(S-ID) use statistics to approximate the shape of the data distribution to compare center and spread of two or more different data sets.</td>
<td>2. Exploring Data: Fitting a Line to Data</td>
</tr>
<tr>
<td>(S-ID) interpret differences in shape, center, and spread in the context of the data sets, accounting for possible effects of extreme data points.</td>
<td>3. Exploring Data: Measures of Central Tendency</td>
</tr>
<tr>
<td>(S-ID) summarize categorical data for two categories in two-way frequency tables.</td>
<td>4. Exploring Data: Measures of Dispersion</td>
</tr>
<tr>
<td>(S-ID) represent data on two quantitative variables on a scatter plot, and describe how the variables are related.</td>
<td>5. Introduction to Probability</td>
</tr>
<tr>
<td>(S-ID) fit a function to the data; use functions fitted to data to solve problems in the context of the data.</td>
<td>6. Counting Methods: Permutations</td>
</tr>
<tr>
<td>(S-ID) interpret the slope and the intercept of a linear model in the context of the data.</td>
<td>7. Counting Methods: Combinations</td>
</tr>
<tr>
<td>(S-ID) compute (using technology) and interpret the correlation coefficient of a linear fit.</td>
<td>8. Probability, Unions, and Intersections</td>
</tr>
<tr>
<td>(S-CP) describe events as subsets of a sample space using characteristics of the outcomes, or as unions, intersections, or complements of other events.</td>
<td>9. Independent Events</td>
</tr>
<tr>
<td>(S-CP) understand that two events A and B are independent if the probability of A and B occurring together is the product of their probabilities, and use this characterization to determine if they are independent.</td>
<td><strong>Sample Assessments:</strong></td>
</tr>
<tr>
<td>(S-CP) use permutations and combinations to compute probabilities of compound events and solve problems.</td>
<td>Ex1: How many different license plates can be made if 7 characters must be used, any letter or number is allowed in any position, but no character can be used more than once?</td>
</tr>
<tr>
<td><strong>Projects:</strong></td>
<td>Ex2: Using the data from the 2010 US Census [not shown here], create a histogram, box and whisker plot, and stem and leaf plot of each state’s total population. Use each graph to describe the center and spread of the distribution. How do these graphs display the same set of data differently?</td>
</tr>
<tr>
<td>• Students will create a simple statistical analysis of a set of data of their choice in the social sciences or</td>
<td></td>
</tr>
</tbody>
</table>
The data should have at least 40 elements. Students will:
- visually represent the data using a histogram, stem and leaf plot, and a box and whisker plot
- numerically summarize the data by finding the mean and standard deviation, and the median and five-number summary.
- summarize their findings from both the graphs and computations. Students should compare and contrast their results, as well as describe which graphic and numeric summaries are most appropriate and why.

- Students will investigate the effect of outliers on a set of data. Students will take a set of data that is approximately “normally distributed” and create a histogram and numeric summary. Then, students will add a single extreme outlier, and repeat the computations. Students should present their results using a poster or Microsoft Powerpoint.

Instructional Strategies:
Interdisciplinary Connections

Students will explore how data and probability can be used to make decisions and predictions in the social and physical sciences.

Technology Integration

- Technology (TI-83 graphing calculators, Geogebra, Mathematica software, and various web applets) will be used to simulate random behavior, generate graphs of data, and quickly find measures of central tendency and spread.
Students can use Fathom software to easily generate graphs and numeric summaries of data.

The applet above demonstrates the effect that outliers can have on the mean and median. Students can use their own data and observe the results of modifying a single element.

Media Literacy

Students will be asked to scour media sources for examples of statistics in the news. Students will be encouraged to examine these studies for potential biases. Through this activity, the unit can be extended to briefly discuss sampling techniques.

Global Perspectives

Real data that compares the growth rates of different nations’ economies and populations will be used to motivate statistics topics.

The following skills and themes listed to the right should be reflected in the design of units and lessons for this course or content area.

21st Century Skills:
- Creativity and Innovation
- Critical Thinking and Problem Solving
- Communication and Collaboration
- Information Literacy
- Media Literacy
- Life and Career Skills

21st Century Themes (as applies to content area):
<table>
<thead>
<tr>
<th>Financial, Economic, Business, and Entrepreneurial Literacy</th>
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<tbody>
<tr>
<td>Civic Literacy</td>
</tr>
<tr>
<td>Health Literacy</td>
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Texts and Resources:
